Derived orders and Auslander–Reiten-quivers

based on the part of the talk by Wolfgang Rump (Eichstätt)

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Let Ω be a poset and F a skewfield. By denote by $\mathrm{Rep}_F(\Omega)$ the category of representations of Ω over F that is the the category of all systems $X = (X, X_i)_{i\in\Omega}$, where X is the finite dimensional vector space over F and for each $i \in \Omega$ X_i is the subspace of X such that, if $i \leq j$ then $X_i \subset X_j$.

Assume that p is a minimal element in Ω and q is a maximal element in Ω such that $p \not\leq q$. Let C be the set of all elements $c \in \Omega$ such that $p \not\leq x \not\leq q$. If C is a chain then we can associate to Ω its derivative Ω' of Ω in the following way. The elements of Ω' are given by $\Omega \setminus C \cup C^+ \cup C^-$, were $C^\pm := \{c^\pm \mid c \in C\}$. The relation $\leq$ is obtained in a natural way from the order in Ω and the relations $p \leq c^+$, $c^- \leq c^+$, $c^- \leq q$ for each $c \in C$. Let $X = (X, X_i)$ be a representation of Ω. We define derived representation X' the rule $X'_{c^+} = X_c + X_p$ and $X'_{c^-} = X_c \cap X_q$. Let $B = (F, F_i)$ be given by the $F_i = F$ if $p \geq i$ and $F_i = 0$ otherwise.

Theorem (Zavadskij)**.** *There is a surjection between the isomorphism classes of indecomposable representations of Ω and isomorphism classes of indecomposable representations of Ω' which is one to one to one except fiber corresponding to B which is finite.*

Let R complete discrete valuation domain. Denote by K its fraction ring and by Π its radical. Λ is called an R-order provided Λ is an R-algebra which is finitely generated and free as an R-module. By Λ-lattice we mean a finitely generated Λ-module. Tiled R-order is $\Lambda = (\Pi^{e_{i,j}}) \subset M_n(K) = A$. Let S be the unique simple A-module. We consider the infinite poset $\mathfrak{P}_\Lambda$ of all nonzero and projective Λ-submodules of S.