QUASIHEREDITARY ALGEBRAS ASSOCIATED WITH REDUCED WORDS IN COXETER GROUPS

BASED ON THE TALK BY IDUN REITEN

The talk was based on a joint work with Osamu Iyama.

Throughout the talk k is a fixed algebraically closed field.

§1. Quasihereditary algebras

Throughout this section we fix a finite dimensional k-algebra Λ together with a complete sequence $(S_1, \ldots, S_n)$ of pairwise nonisomorphic simple Λ-modules.

Notation.
For $i \in [1, n]$ we denote by P_i the projective cover of i.

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For $i \in [1, n]$ we denote by Δ_i the largest factor module of P_i with composition factors $S_1, \ldots, S_i$. We call $\Delta_1, \ldots, \Delta_n$ *standard modules*.

Notation.
We denote by $\mathscr{F}(\Delta)$ the class of Λ-modules which have filtrations using $\Delta_1, \ldots, \Delta_n$.

Definition.
We say that Λ is *quasihereditary* if $\operatorname{End}_\Lambda(\Delta_i) = k$ and $P_i \in \mathscr{F}(\Delta)$ for each $i \in [1, n]$.

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We say that Λ is *strongly quasihereditary* if Λ is quasihereditary and $\operatorname{pd}_\Lambda \Delta_i \leq 1$ for each $i \in [1, n]$.

§2. Construction of quasihereditary algebras

Proposition.
Let $\mathscr{C}$ be an extension closed Hom-finite subcategory of an abelian category. Let $T := \bigoplus_{i \in [1,n]}$ for pairwise nonisomorphic indecomposable objects of $\mathscr{C}$ and $\Gamma := \operatorname{End}(T)$. If $\operatorname{Ext}^1(T, T) = 0$, $\operatorname{gl.dim} \Gamma < \infty$, and the minimal left $\operatorname{add}(\bigoplus_{j \in [1, i-1]} T_j)$-approximation $T_i \xrightarrow{f_i} T_i'$ of T_i is surjective for each $i \in [1, n]$, then Γ is strongly quasihereditary with the standard modules $\operatorname{Hom}(\operatorname{Ker} f_i, T)$. Moreover, if T_i' is either indecomposable or zero for each $i \in [1, n]$, then each indecomposable projective Γ-module has a unique Δ-composition series.

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For the rest of the section we assume that Q is a finite quiver without oriented cycles and we denote by Λ the associated preprojective algebra. Finally, we assume that $Q_0 = [1, n]$.

NOTATION.

For $i \in [1, n]$ we put

$$I_i := \Lambda(1 - e_i)\Lambda,$$

where e_i denotes the corresponding idempotent in Λ.

DEFINITION.

By the *Coxeter group* W of Q we mean the group generated by s_i, $i \in [1, n]$, together with the following relations:

- $s_i^2 = 1$, $i \in [1, n]$,
- $s_i s_j = s_j s_i$, $i, j \in [1, n]$, there is no arrow between i and j,
- $s_i s_j s_i = s_j s_i s_j$, $i, j \in [1, n]$, there is exactly one arrow between i and j.

DEFINITION.

By a *reduced expression* of $w \in W$ we mean every sequence $(i_1, \ldots, i_t)$ of vertices of Q such that $w = s_{i_1} \cdots s_{i_t}$ and $t \leq l$ for each sequence $(j_1, \ldots, j_l)$ such that $w = s_{j_1} \cdots s_{j_l}$.

For the rest of the section we fix $w \in W$ together with a reduced expression $(i_1, \ldots, i_t)$.

NOTATION.

We put

$$\Lambda_w := \Lambda/(I_{i_1} \cdots I_{i_t}) \qquad \text{and} \qquad T := \bigoplus_{j \in [1,t]} P_{i_j}/I_{i_1} \cdots I_{i_j} P_{i_j}.$$

THEOREM.

The algebra $\operatorname{End}_{\Lambda_w}(T)$ is strongly quasihereditary.