DERIVED EQUIVALENCE CLASSIFICATION OF THE GENTLE ALGEBRAS ARISING FROM SURFACE TRIANGULATIONS

BASED ON THE TALK BY SEFI LADKANI

Let S be a compact connected oriented Riemann surface with non-empty boundary. We denote by g the genus of S. Moreover, let X_1, $\dots$, X_b be the connected components of ∂S. We also fix a finite subset M of ∂S. For each $i \in \{1, \dots, b\}$ we assume that $M \cap X_i \neq \varnothing$ and we denote by n_i the number of the elements of $M \cap X_i$. By a triangulation we mean every maximal collection of compatible arcs with end points in M. A triangle of such a triangulation is called internal if all its sides are arcs. On the other side, we call a triangle external if two of its sides are contained in M. For $i \in \{1, \dots, b\}$ we denote by d_i the number of the external triangles having non-empty intersection with X_i. The sequence $(g, b, n_1, d_1, \dots, n_b, d_b)$ is called the parameter sequence of a triangulation.

Given a triangulation T we may associate with T a quiver with mutation (Q_T, W_T) in the following way. The vertices of Q_T are the arcs of T. If i and j are vertices of T, then we have an arrow $i \to j$ if and only i and j have a common point and one goes from i to j in the clockwise direction. Finally, W_T is the sum over the internal triangles of the compositions of the corresponding arrows. One shows that the Jacobian algebra associated with (Q_T, W_T) is a gentle algebra. We also note that flips of triangulations correspond to quiver mutations. We say that a mutation of (Q_T, W_T) is good if the corresponding Jacobian algebras are derived equivalent.

Theorem. *For triangulations T and T' the following conditions are equivalent.*

(1) *T and T' have the same parameter sequences.*
(2) *Λ_T and $\Lambda_{T'}$ have the invariant of Avella-Alaminos and Geiss.*
(3) *Λ_T and $\Lambda_{T'}$ are derived equivalent.*
(4) *(Q_T, W_T) and $(Q_{T'}, W_{T'})$ can be connected by a sequence of good mutations.*

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