THE INTEGRAL CLUSTER CATEGORY

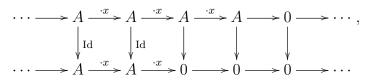
BASED ON THE TALK BY SARAH SCHEROTZKE

For an autoequivalence F of an additive category \mathcal{A} we define the orbit category \mathcal{A}/F in the following way. The objects of \mathcal{A}/F coincide with the objects of \mathcal{A} , however if X and Y are objects of \mathcal{A} , then

$$\operatorname{Hom}_{\mathcal{A}/F}(X,Y) := \bigoplus_{n \in \mathbb{Z}} \operatorname{Hom}_{\mathcal{A}}(X,F^{n}Y).$$

The canonical functor $\pi : \mathcal{A} \to \mathcal{A}/F$ is universal among the functors $G : \mathcal{A} \to \mathcal{B}$ such that $G \circ F \simeq G$.

The following example show that \mathcal{A}/F does not have to be a triangulated category even if \mathcal{A} is a triangulated category and F is an exact functor. Namely, let \mathcal{A} be the bounded derived category of the modules over the algebra $k[x]/x^2$ and F := [2]. If μ is the following map



then $\operatorname{End}_{\mathcal{A}/F}(k) = k[\mu]$. On the other hand, μ is an monomorphism, i.e. if $\mu \circ f = 0$, then f = 0. However, in triangulated categories monomorphisms split, which leads to a contradiction.